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A Macroscopic Analogue of The Nuclear Pairing Potential

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Abstract
A macroscopic system involving permanent magnets is used as an analogue to nucleons in a nucleus to illustrate the significance of the pairing interaction. This illustrates that the view of the total nuclear energy based only on the nucleon occupancy of the energy levels can yield erroneous results and it is only when the pairing interaction is considered that a correct picture emerges. The macroscopic analogue of combined gravitational and magnetic interactions shows that a consideration of only the gravitational interaction can lead to an incorrect picture and that the inclusion of magnetic coupling is needed to properly describe the ordering of the energy levels.

Keywords: Nuclear pairing energy, nuclear structure, nuclear energy levels, excited states

Introduction

The shell model has been very successful in describing the ground state spin and parity, J^{π}, of large number of stable and unstable nuclei (Dunlap, 2004). The model is also sometimes effective in explaining excited state spins and parities. This is particularly the case for odd-even and even-odd nuclei (where this designation refers to the number of neutrons, N, and protons, Z, respectively) where excited states are often described on the basis of nucleon excitations and the corresponding nucleon configurations give rise to the resulting spin properties. It is, however, not always easy to relate nucleon configurations to the resulting energy levels because of the strong nucleon pairing interaction. The importance of the pairing interaction sometimes makes the nucleon configuration that corresponds to particular energy levels counterintuitive. Some even-even nuclei are particularly useful in providing insight into this phenomenon.

Nuclear energy levels of ^{90}Zr

Nearly all even-even nuclei have a first excited state with $J^{\pi} = 2^{+}$ resulting from quantized rotational energy (Scharff-Goldhaber, 1953). However, a few anomalous even-even nuclei do not have a rotational first excited state. Most of these are doubly magic nuclei (e.g. ^{16}O, ^{40}Ca, ^{208}Pb) (Hirao *et al.*, 1958)) which are highly spherically symmetric and hence have a low moment of inertia giving rise to a large energy associated with the lowest level rotational state. In such cases nucleon excitation modes can be responsible for the lowest lying excited states.

An interesting illustrative example is ^{90}Zr, with $(N,Z) = (50,40)$ (Kloepper *et al.*, 1959)). A simplified energy level diagram for ^{90}Zr is shown in Figure 1 (Firestone *et al.,* 1996). Although only singularly magic this nuclide shows behavior similar to doubly magic nuclei by having a 0^{+} first excited state. The nucleon configuration for this excited state can be explained by starting with the ground state configuration for ^{90}Zr. The magic number of 50 neutrons gives rise to filled shells where all states up to (and including) $1g_{9/2}$ are filled (Dunlap, 2004)). The 40 protons have a ground state configuration of $(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^6(1d_{3/2})^4(2s_{1/2})^2(1f_{7/2})^8(1f_{5/2})^6(2p_{3/2})^4(2p_{1/2})^2(1g_{9/2})^0$ corresponding to a partially filled shell.

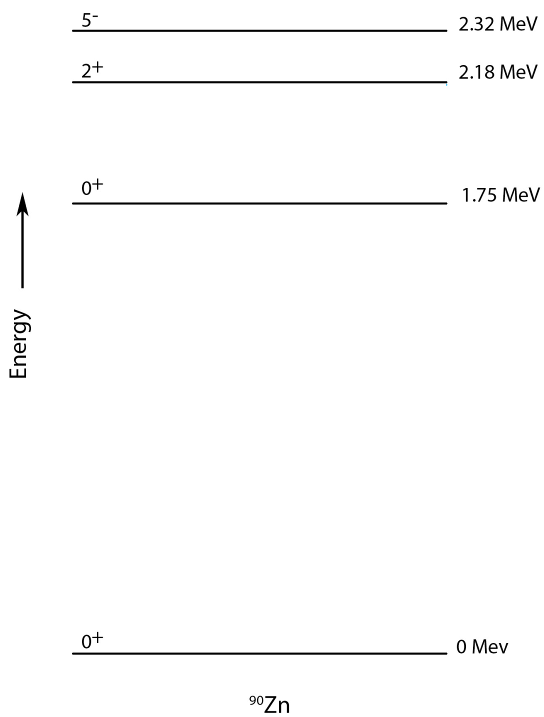

Figure 1: Low lying excited states of a ^{90}Zr nucleus (adapted from Firestone et al. (1996)).

The simplest nucleon excitation would be to excite one $2p_{1/2}$ proton into the $1g_{9/2}$ state. This would give a resulting nuclear spin in the range of $4 \leq J \leq 5$ (that is between $J = 9/2 - 1/2$ and $J = 9/2 + 1/2$). The parity will be odd as a result of the combination of the odd parity p state and the even parity g state. This would not explain the lowest lying 0^+ excited state at 1.75 MeV but can explain the 5^- state at higher energy. The intermediate 2^+ excited state is a rotational state as observed in nearly all even-even nuclei as described above. This leaves the 0^+ state to be explained. The 0^+ state would be consistent with moving two $2p_{1/2}$ protons into the $1g_{9/2}$ state. This, in fact is the appropriate proton configuration for this energy level and shows that the total energy of the nucleus is increased less by moving two protons from the ground state to a higher level than by moving only one proton from the ground state to a higher energy level. This (perhaps counterintuitive) behavior is explained by the proximity of the $2p_{1/2}$ and $1g_{9/2}$ levels combined with the presence of a strong nuclear pairing potential. It requires less energy to move two protons to a higher energy level that it does to break the pairing interaction and move only one.

Macroscopic analogue for the nuclear paring energy

The concept of nuclear energy levels can be made more tractable to students by considering a macroscopic analogue. Consider two identical masses in a gravitational potential. In a two level system the masses may be in their ground state (both in the lowest energy level), in the first "excited" state (one in the lowest level and one in the higher level) or the second "excited" state (both masses in the higher level). These configurations are illustrated in Figure 2. The change in energy of the system is given by the work done against the gravitational force, *mg*, in changing the height of the mass;

$$\Delta E = \int F dh = mgh \qquad (1)$$

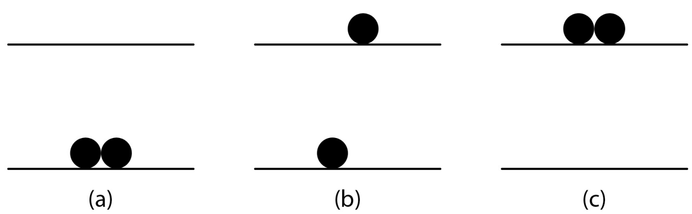

Figure 2: Ground state and first two excited states for a system of two masses and two levels in a gravitational potential.

Thus, for a ground state energy defined as 0, the first excited state will have energy *mgh* and the second excited state will have energy 2*mgh*. To make this example more quantitative, consider two masses of 0.02 kg and a step of height 0.10 m. The energy level diagram for the system is shown in Figure 3.

Magnetic interactions between permanent magnetic may be viewed as a macroscopic analogue of the nuclear pairing potential. Consider the two masses in Figure 2 as permanent magnetic that, are attracted to each other in the ground state. In order to move one magnet to a higher (gravitational) level as in Figure 2(b) the work required will be the total of the work required to separate the two magnets plus the work to raise the mass a height *h*. That is, the change in energy of the system will be

$$\Delta E = W_{mag} + mgh \qquad (2)$$

If the second magnet is now raised to the higher level and "paired" with the first magnet (Figure 2(c)) then the change in the energy of the system will be the work required to raise the magnet, *mgh*, reduced by the magnetic interaction energy that is recovered when the magnets are stuck together;

$$\Delta E = (mgh - W_{mag}) \qquad (3)$$

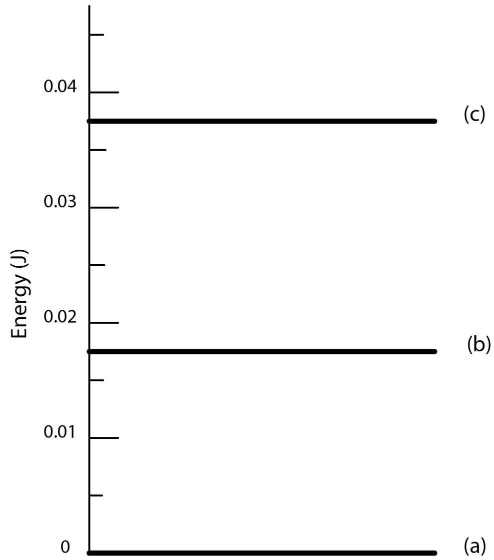

Figure 3: Energy level diagram for two 0.02 kg masses in a gravitational potential with two levels separated by 0.10 m. The level designations ((a), (b) and (c)) correspond to the configurations shown in Figure 2.

This may be combined with equation (2) to give the total change in energy between the ground state (Figure 2(a)) and the state in Figure 2(c) as

$$\Delta E = (W_{mag} + mgh) + (mgh - W_{mag}) = 2mgh \qquad (4)$$

That is, Figures 2(a) and 2(c) represent a change in gravitational potential ($2mgh$) but no change in magnetic state. To quantify this example, the actual magnetic interaction energy for the two magnets may be determined by a simple experiment. Figure 4 shows the use of a calibrated strain gauge to measure the force between to magnets as they are pulled apart. The measured force between two 0.020 kg spherical ferrite magnets (of diameter d = 0.025 m) is shown in Figure 5 as a function of the center-to-center distance, x, between the magnets.

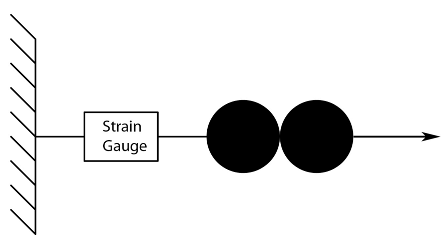

Figure 4: Diagram of the experiment for measuring the force between to magnets.

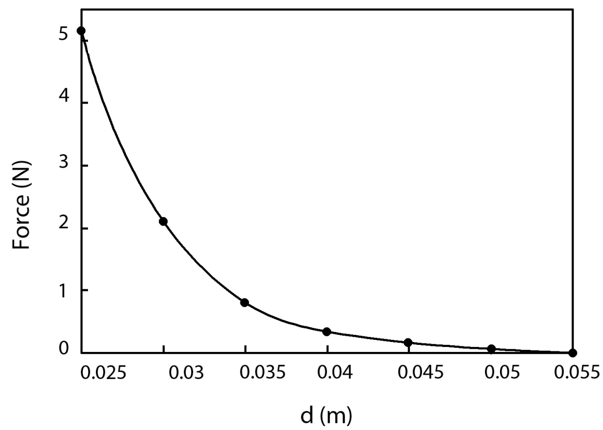

Figure 5: Force as a function of center-to-center distance between two 0.025 m diameter spherical magnets as measured by the method in Figure 4. (Note that the force is ~ 0 N for x > 0.055 m.)

The total work required to separate the magnets is the integral of the force from $x = d$ (when the magnets are in contact) to infinity as;

$$W_{mag} = \int_{d}^{\infty} F(x)dx \qquad (5)$$

The total work to separate the magnets as calculated from equation (5) from the data in Figure 5 is found to be 0.032 J. The energy levels corresponding to the three configurations of the magnetic masses as shown in Figure 2 can readily be found to be 0 J for the ground state [Figure 2(a)], $2mgh = 2\times(0.02\text{ kg})\times(9.8\text{ m}\cdot\text{s}^{-2})\times(0.1\text{ m}) = 0.037$ J for the first excited state [Figure 2(c)] and $mgh + W_{mag} = (0.02\text{ kg})\times(9.8\text{ m}\cdot\text{s}^{-2})\times(0.1\text{ m}) + (0.032\text{ J}) = 0.051$ J for the second excited state [Figure 2(b)]. These levels are illustrated in Figure 6.

Conclusion

Introductory nuclear physics courses often consider the properties of excited nuclear states in the context of the nuclear shell model. It is most common to consider odd-even or even-odd nuclei, which consist of a doubly magic core with one additional nucleon. These nuclei have low-lying excited states that are generally well described by the extreme single nucleon model where the spin and parity is the result of a single neutron or proton excitation. In this context, Williams (1991) has provided a detailed description of the excited states of the odd-even nucleus ^{17}O (see p. 151). Krane (1988) has also discussed ^{17}O, as well as its even-odd mirror nucleus ^{17}F (see p. 131-132). Dunlap (2004) has followed along these lines and has considered the odd-even nucleus ^{41}Ca (see p. 65-66). A detailed analysis of the higher energy excited states of these nuclei often requires a consideration of nuclear pairing energy and the formation of nucleon

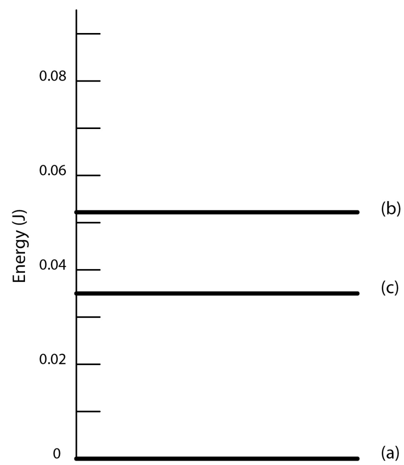

Figure 6: Energy level diagram for two 0.02 kg magnetic masses in a gravitational potential with two levels separated by a vertical distance of 0.1 m. The level designations ((a), (b) and (c)) correspond to the configurations shown in Figure 2.

configurations that are not single nucleon states. This situation is similar to nucleon excitations and the formation of excited states in even-even. The excited states of ^{90}Zr provide a simple example where the nuclear pairing interaction is necessary to explain the energies of even the lowest-lying excited states.

The macroscopic analogue as described above illustrates that the total change in the energy of a system must take into account all interactions present, in this case gravitational and magnetic, and that the ordering of the energy levels may seem counterintuitive if one considers only one of the relevant interactions. This analogue may be extended to nuclear energy levels where the pairing interaction takes the place of the magnetic interaction. This analogy can be conveniently integrated into the discussion of excited nuclear states in the context of the shell model and can readily include a classroom demonstration that illustrates the principles behind the formation of energy levels as illustrated in Figure 6.

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